

Examining the Sensitivity of Ice Sheet Models to Updates in Rheology (n=4)

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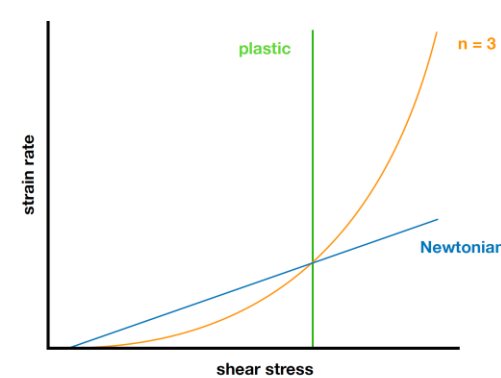
The Big Picture (TLDR...)

In ice sheet modeling, glacial ice is usually assumed to be described by a power-law rheology (Glen'). Historically, the value of n in this formula was taken to be 3. However, recent work has suggested n=4 is a better fit with observations. **Our goal:** Using an idealized ice sheet, investigate the potential impact of this on existing simulations, which have generally assumed n=3.

Glen's Law and the value of n

Ice sheet models generally assume ice to be a non-Newtonian fluid with a shear-thinning rheology described by Glen's Law, in which the ice viscosity is given by a power-law relationship with the strain-rate invariant:

- $\dot{\epsilon} = A\tau^n$, where
 - $\dot{\epsilon}$ is the strain rate invariant
 - $A(T)$ is the rate factor
 - τ is the deviatoric stress
 - n is the Glen's law exponent.



n=3? n=4?

n has generally been taken to be 3 in most glaciological modeling applications. However, recent work (Millstein et al, 2022) has suggested that n=4 better matches Antarctic observations, implying a greater degree of nonlinearity than previously thought.

What does that mean?

Much of the ice sheet modeling effort to date has been performed using n=3. The impact of this potential mismatch on existing ice-sheet modeling projections is an open question.

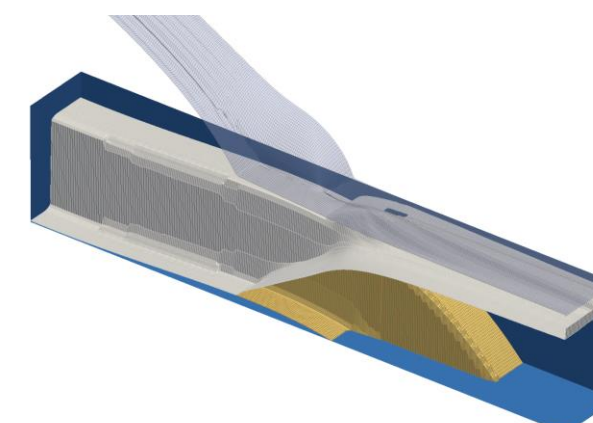
Our Experiment

In most cases, models are initialized to match ice sheet observations of thickness and velocity. An incorrect value of n implies that this process will attempt to match an n=3 configuration to these observations (in which the actual value is n=4). To examine this impact, we do the following experiment:

- Set up an idealized ice sheet with n=4
- Perform our standard inversion process with n=3 to match the "observed" n=4 thickness and velocity fields.
- Force both n=3 and n=4 configurations in a similar way to determine the differences in response (if any) with our assumption of n=3.

Idealized Experiment – MISMIP+

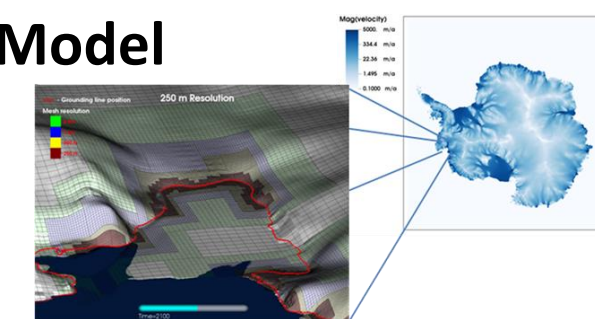
- The MISMIP+ experiment [2], is a marine ice sheet in a channel with a retrograde bed section, designed to highlight ice shelf buttressing effects.
- Parameters like surface mass balance and basal friction are chosen to place the steady-state grounding line on the retrograde slope due to the impact of ice shelf buttressing.
- The rate factor must be rescaled for n=4. To do this, we pick a reference stress (in our case, 100 kPa, and equalize the n=3 and n=4 viscosities. This results in a value for A for n=4 which is 4 orders of magnitude smaller than the n=3 value.
- In the evolution experiment, a prescribed subshelf melt weakens the ice shelf, causing thinning and retreat for 100 years. After 100 years, the perturbation is removed, allowing recovery.



MISMIP+ cross-section showing channel walls (brown) and steady-state ice upper- and lower-surfaces.

Numerical Model

- We use the BISICLES ice sheet model [3], an adaptive mesh refinement (AMR) ice sheet model which dynamically adds refined meshes where needed to assure solution accuracy.
- Inversion performed using the Tikhonov-penalized nonlinear CG approach described in [4].
- The inversion infers a basal friction field along with a viscosity multiplier to represent the impact of damage, rheology, etc.



BISICLES-computed Antarctic ice velocity field. Inset shows adaptive meshing near the Pine Island Glacier grounding line.

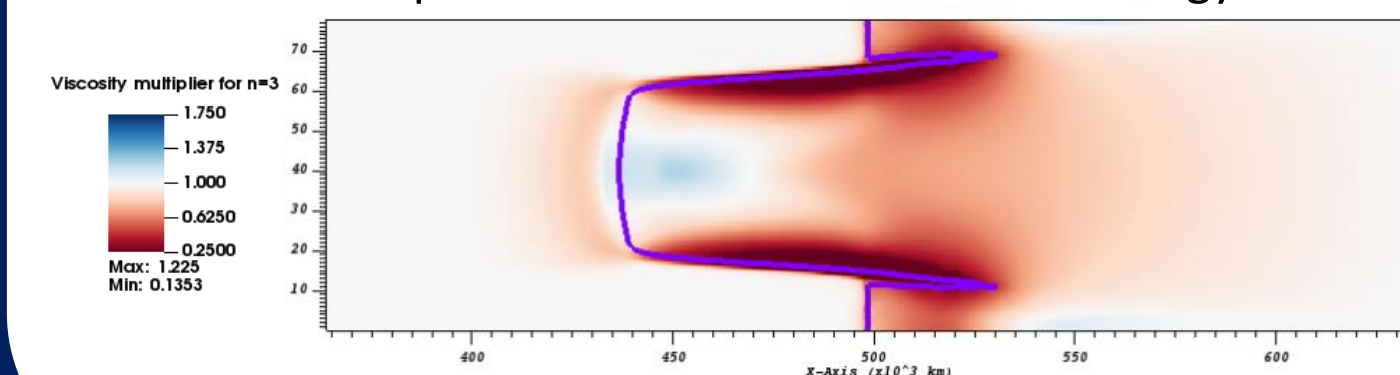
References

- [1] Millstein, J.D., Minchew, B.M. & Pegler, S.S. Ice viscosity is more sensitive to stress than commonly assumed. *Commun Earth Environ* 3, 57 (2022). <https://doi.org/10.1038/s43247-022-00385-x>
- [2] Asay-Davis, X. S., et al: Experimental design for three interrelated marine ice sheet and ocean model intercomparison projects: MISMIP v. 3 (MISMIP +), ISOMIP v. 2 (ISOMIP +) and MISOMIP v. 1 (MISOMIP1), *Geosci. Model Dev.*, 9, 2471–2497, <https://doi.org/10.5194/gmd-9-2471-2016>, 2016.
- [3] <https://bisicles.lbl.gov>
- [4] Cornford, S.L. et al: Century-scale simulations of the response of the West Antarctic Ice Sheet to a warming climate, *The Cryosphere*, 9, 1579–1600, <https://doi.org/10.5194/tc-9-1579-2015>, 2015.

n=3 Inversion to match n=4

We match "observed" ice thickness and velocity field (the steady-state n=4 solution) by optimizing basal friction and a viscosity multiplier, representing the impact of "damaged" ice, heating, etc.

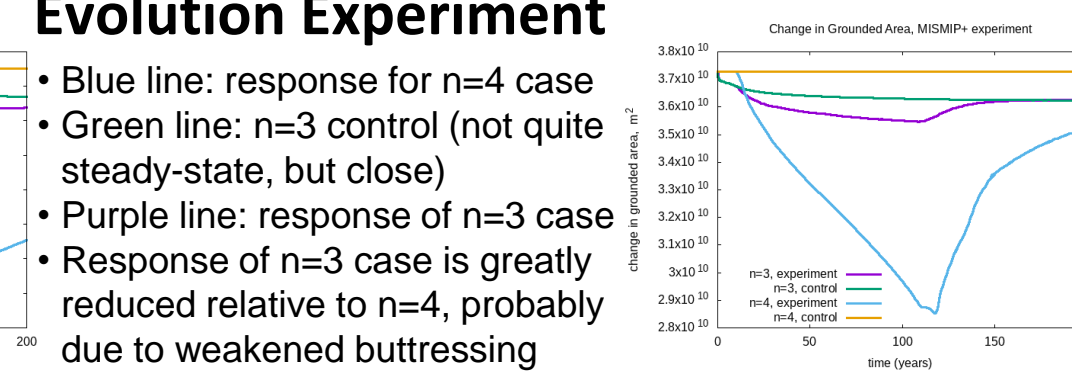
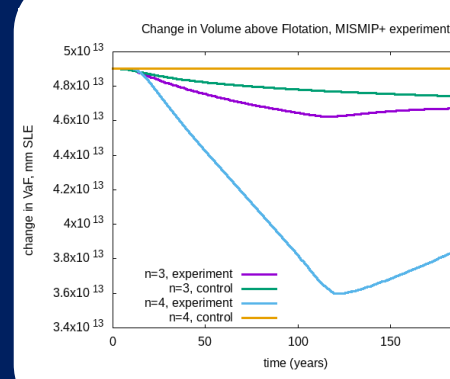
- Basal friction coefficients remain identical, as expected since the mismatch results from differences in rheology.
- **The viscosity multiplier shows the impact of rheology – substantial softening (reduced viscosity) in regions with higher shear to compensate for the difference in rheology.**



Inversion-computed viscosity multiplier – red represents softening of the ice in higher-shear regions to match n=4 steady-state.

Evolution Experiment

- Blue line: response for n=4 case
- Green line: n=3 control (not quite steady-state, but close)
- Purple line: response of n=3 case
- Response of n=3 case is greatly reduced relative to n=4, probably due to weakened buttressing from the viscosity multiplier.



Conclusions

- Impact of mismatched rheology appears in inversion results in the form of prescribed softening to match the n=4 observed results.
- *Much of what is currently attributed to "damage" in inversions & initializations may be compensating for the incorrect value for n.*
- Compounding this, ice sheet models which compute such viscosity multipliers (including BISICLES) often keep them fixed in time, unless attempting to evolve a "damage" field.
- Dynamic response also affected – impact of ice shelf weakening and changes in buttressing appears to be greatly reduced.