

3D convection, phase change, and solute transport in mushy sea ice

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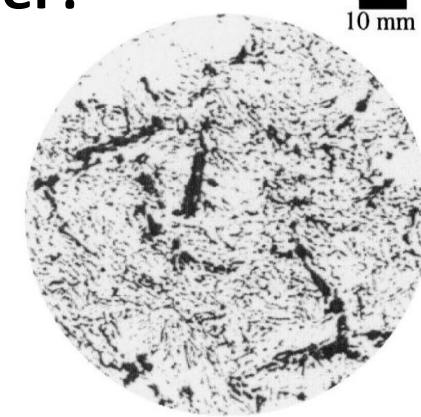
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Motivation

Sea ice is a mushy layer of ice crystals and brine. Dense brine drains during ice formation, while some brine is trapped within sea ice. Observations (Fig. 1) and 1- and 2-D simulations suggest that warming sea ice may release some of this brine. **Our goal:** investigate this mechanism using 3-D numerical simulations

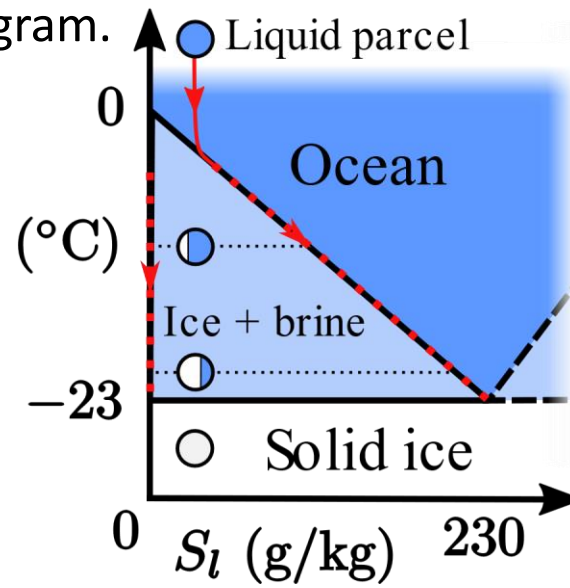
What is a Mushy Layer?

(right) Sea ice is a porous mixture of solid ice crystals (white) and liquid brine (dark) [2].



(lower) Trajectory (→) of a solidifying salt water parcel through the phase diagram.

As the temperature T decreases, more ice forms and the residual brine salinity S_l increases making the uid denser, which can drive convection. Using a linear approximation for the liquidus curve, the freezing point is $T_f(S_l) = 0.1S_l$.



Governing Equations

Continuous equations for conservation of momentum (1), mass (2), salt (3) and energy (4) are found by averaging over lengths greater than the pore scale of sea ice [4, 5].

1. $\vec{U} = -\frac{k_w(\chi)}{\eta}(\nabla p - \rho_l \vec{g})$
2. $\nabla \cdot \vec{U} = 0$
3. $\frac{\partial S}{\partial t} = \vec{U} \cdot \nabla S_l = \nabla \cdot \chi D_l \nabla S_l$
4. $\frac{\partial H}{\partial t} + \rho_0 c_{p,l} \vec{U} \cdot \nabla T = \nabla \cdot [k_l \chi + (1 - \chi)k_s] \nabla T$

\vec{U} (Darcy Velocity), χ (porosity), p (pressure), T (temperature), S_l (liquid salinity), $S = \chi S_l$ (bulk salinity), η (viscosity), D_l (salt diffusivity), α, β (thermal, haline expansion), $c_{p,l}, c_{p,s}$ (liquid/solid specific heat), k_l, k_s (liquid, solid heat conductivity), K_0 (reference permeability)

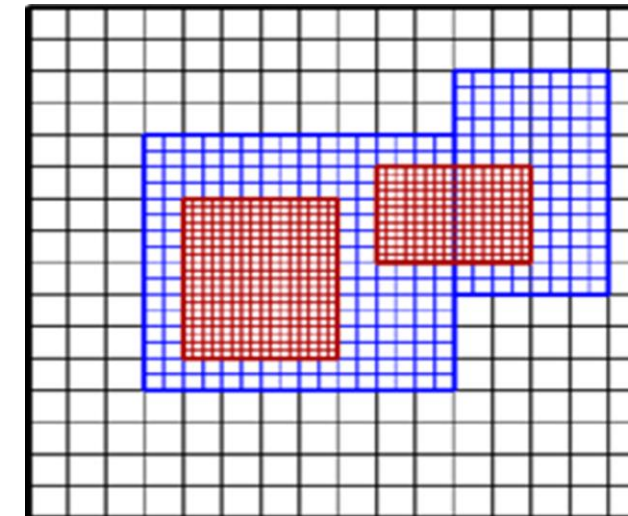
$$H = \rho_0 \{L\chi + [\chi c_{p,l} + (1 - \chi)c_{p,s}]T\} \text{ (enthalpy)}$$

$$\rho_l = \rho_0 [1 - \alpha T + \beta S_l] \text{ (liquid density)}$$

$$K(\chi)^{-1} = (a^2/12)^{-1} + [K_0 \chi^3 / (1 - \chi)^2]^{-1} \text{ (permeability)}$$

Adaptive Mesh Refinement

- Fine mesh resolution is required to resolve brine channels, but don't know *a priori* where they will be.
- We use *Block-structured adaptive mesh refinement (AMR)*, in which we locally and dynamically refine the computational mesh where needed.
- Our implementation is built upon the Chombo AMR library. (<http://chombo.lbl.gov>)



ABOVE: Sample AMR meshes – black mesh is base level (0), blue mesh (level 1) is a factor of 2 finer, while red (level 2) is 4 times finer than level 0.

Numerical Scheme

- Solve (1)-(4) using Chombo finite volume toolkit:
 - Momentum and mass: projection method [3].
 - Energy and solute:
 - Advective terms: explicit, 2nd order unsplit Godunov method.
 - Nonlinear diffusive terms: semi implicit, geometric multigrid.
- Timestepping: 2nd-order semi-implicit scheme due to Twizell, Gumel, and Arigu (1996).

References

- [1] K. Widell et al. Geophysical Research Letters 33.12 (2006), pp. 1{5.
- [2] H. Eicken et al. Cold Regions Science and Technology 31.3 (2000), pp. 207{225.
- [3] D. F. Martin et al. Journal of Computational Physics 227.3 (2008).
- [4] M. G. Worster. Journal of Fluid Mechanics 224.-1 (1991), p. 335.
- [5] M. Le Bars et al. Journal of Fluid Mechanics 550.-1 (2006), p. 149.

This work was funded by US DOE, NERC and a travel grant from the Royal Society.

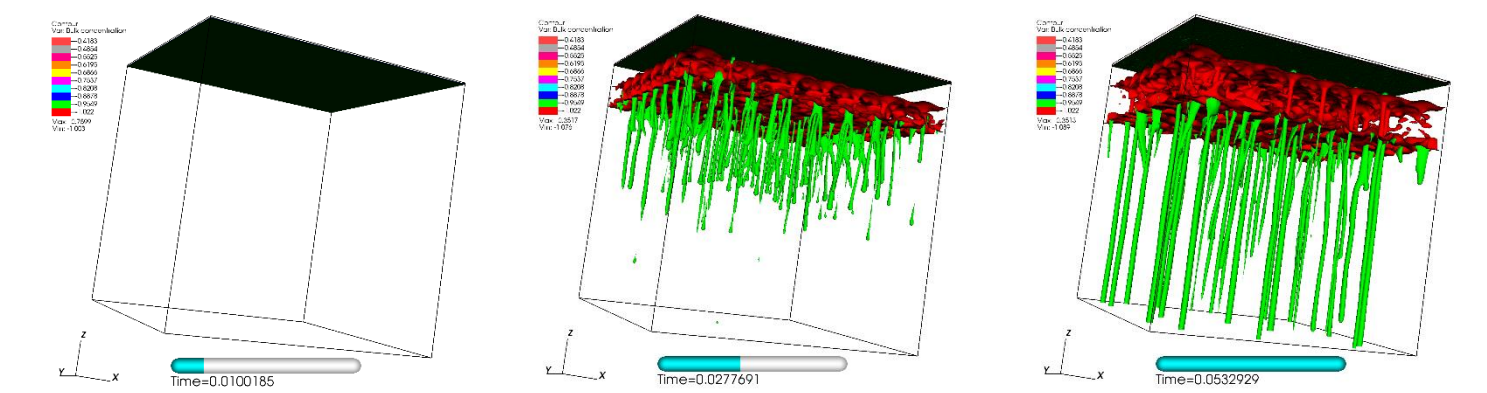
Idealized Experiment

We consider 3-D simulations in a cell of width 0.5m and height 1m. Water of initial salinity $S_0 = 30$ g/kg and temperature $T_f(S_0) + 0.2$ C is initially frozen from above by applying a fixed atmospheric temperature $T_a = -10$ C. We assume $K_0 = 10^{-9} m^2$ and model the underlying water as a porous medium with permeability $K_w = 10K_w$, so we can use Darcy's Law everywhere.

To test the effects of the atmospheric temperature, we also ran the same experiment with varying upper boundary temperatures.

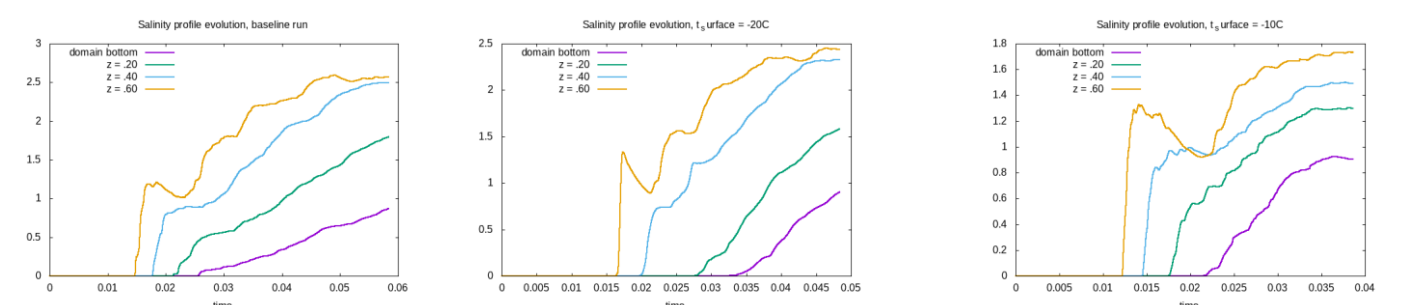
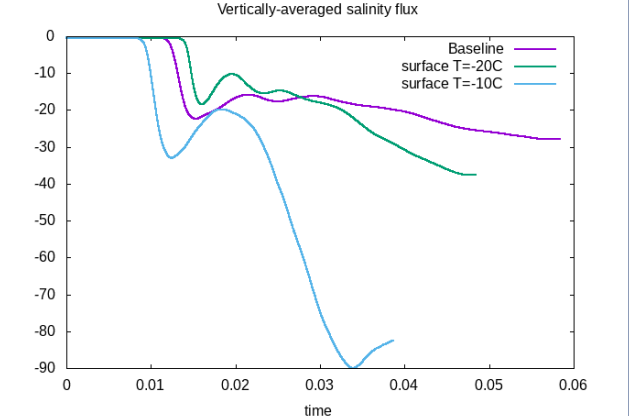
Idealized Experiment Results

As expected, a mushy layer forms on top and brine channels form as the solution evolves.



(above) Nondimensionalized bulk Concentration isosurfaces after 0, 0.027, and 0.05 (time units are dimensionless and scaled by the diffusive timescale)

Right – Vertically averaged salinity flux over time for each experimental run.



Salinity profile evolution at different vertical depths for (left) baseline experiment, (center) $T_a = -20$ C, and (right) $T_a = -10$ C.