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Higher-order versions of Godunov's method have proven highly successful for high-Mach-number compressible flow. One goal of the research being described in this paper is to extend the range of applicability of these methods to more general systems of hyperbolic conservation laws such as magnetohydrodynamics, flow in porous media and finite deformations of elastic-plastic solids. A second goal is to apply Godunov methods to problems involving more complex physical and solution geometries than can be treated on a simple rectangular grid. This requires the introduction of various adaptive methodologies: global moving and body-fitted meshes, local adaptive mesh refinement, and front tracking.

Extension of the Godunov methodology to other systems of equations is more difficult due to the presence of pathologies in the local wave structure which do not appear in gas dynamics. In the applications listed above, one finds that the wave speeds are nonmonotonic functions along their associated wave curves in phase space, requiring more complicated entropy conditions to ensure physically realizable solutions. In addition, there are surfaces in phase space along which the linearized coefficient matrices fail to have a complete set of eigenvectors. We have developed an approximate Riemann problem solver based on a generalization of the Engquist-Osher flux that is suitable for these types of problems [1]. The initial test of the method has been for flow of a multiphase mixture of gas, oil and water in a porous medium. The underlying hyperbolic system consists of three conservation laws for the component densities and exhibits the degeneracies cited above. An additional constraint that the fluid fills the available pore volume leads to an associated parabolic system for pressure and total volumetric flow rate, which appear in the hyperbolic flux as spatially dependent terms. The first example, Fig. 1, shows gravity inversion in a core saturated with pure water in the top third, pure oil in the middle third and pure gas in the bottom third. In addition to the complex frontal structure shown in the figure, there is also a substantial drop in pressure (40%) arising from the mixing of the pure components. The second example, Fig. 2, shows a two-dimensional flow in which water is being injected along the left edge, and oil and gas are being withdrawn along the right edge. Gravity imposes a force in the (-y) direction, so we see some vertical structure in the solution, since the various phases in the problem have different mass densities. In addition to porous media flow this method is also being used to model shock-waves in solids [2] and for compressible magnetohydrodynamics [3].

The other facet of our development of Godunov methods is the introduction of adaptive methodology. To treat more complex flow and problem geometry we have extended the second-order unsplit Godunov method first introduced by Colella [4] to a logically-rectangular moving quadrilateral mesh [5]. This algorithm is based on an upstream-centered predictor-corrector formalism, with the conservative corrector step using finite-volume differencing. The method is second-order accurate for smooth solutions on smooth grids, has a robust treatment of discontinuities with minimal numerical diffusion, and is freestream-preserving. Figure 3 shows a comparison of a numerical solution with experimental data of Zhang and Glass [6] for shock diffraction. A body-fitted quadrilateral grid is generated using a biharmonic solver, so that it is smooth, though not orthogonal. Nonetheless, there is no difficulty resolving this complicated time-dependent flow

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pattern. (See Glass et al [7] for a more detailed comparison.) In figure 4, we show an example of a moving grid calculation, namely, a self-similar ramp shock reflection. At the initial time, an algebraic grid is generated. Then each corner of the grid is given a velocity proportional to its initial location, so that the self-similar reflection pattern approaches a discrete steady state on the moving grid in the limit of long times.

We are also developing other adaptive methods aimed at focusing computational effort where it is needed. In figure 5, we show a result obtained by coupling the adaptive mesh refinement algorithm of Berger and Colella [8] for shock hydrodynamics to a volume-of-fluid type algorithm for tracking an interface between two materials. In the adaptive mesh refinement algorithm, the mesh is locally refined in space and time in response to the appearance of large errors or features in the solution. Refined regions may appear or disappear as a function of time; also, there may be multiple levels of refinement. The multifluid algorithm is coupled to an operator split Godunov method, although operator splitting is not essential. In the approach taken here, only the thermodynamic discontinuity is tracked. There are multiple values for the density and energy for each cell, but only a single velocity per cell, so that any slip that forms along the interface is captured. The figure shows a comparison to experimental data of L. F. Henderson [9] for shock refraction by an oblique material interface. This methodology is being used in a more detailed study of slow-fast refraction [10]. A second application of the multifluid-local refinement algorithm is shown in Fig. 6. The figure contains the result of a calculation of the interaction of a shock with a dense spherical cloud [11]. The calculation was done in cylindrical geometry, with the axis of symmetry located along the left edge of the domain. There are two levels of refinement, each by a factor of four. The finest grids are outlined with boxes in the figure; thus it is apparent that at late times, the finest grids are reserved for the cloud, which in this problem is the region of greatest interest to us. The multifluid tracking eliminates a large class of diffusive errors from the transport of the cloud material. To have performed this calculation on an equivalent uniform grid would have taken 10 times as much CPU time, and 5 times as much memory.

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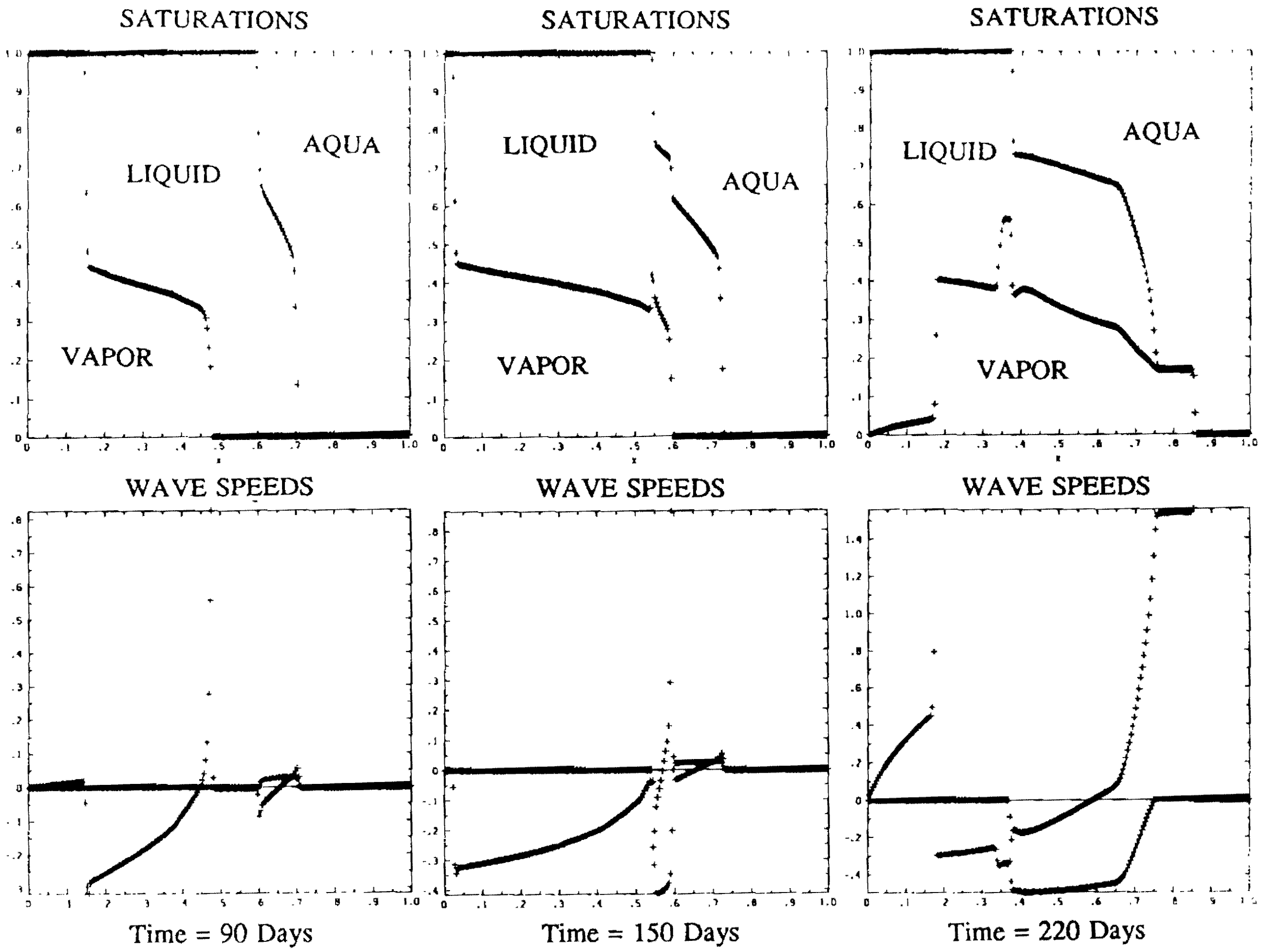


Fig.1. Time history of three component gravity inversion using 240 grid points. Left edge is the bottom; right edge is the top.

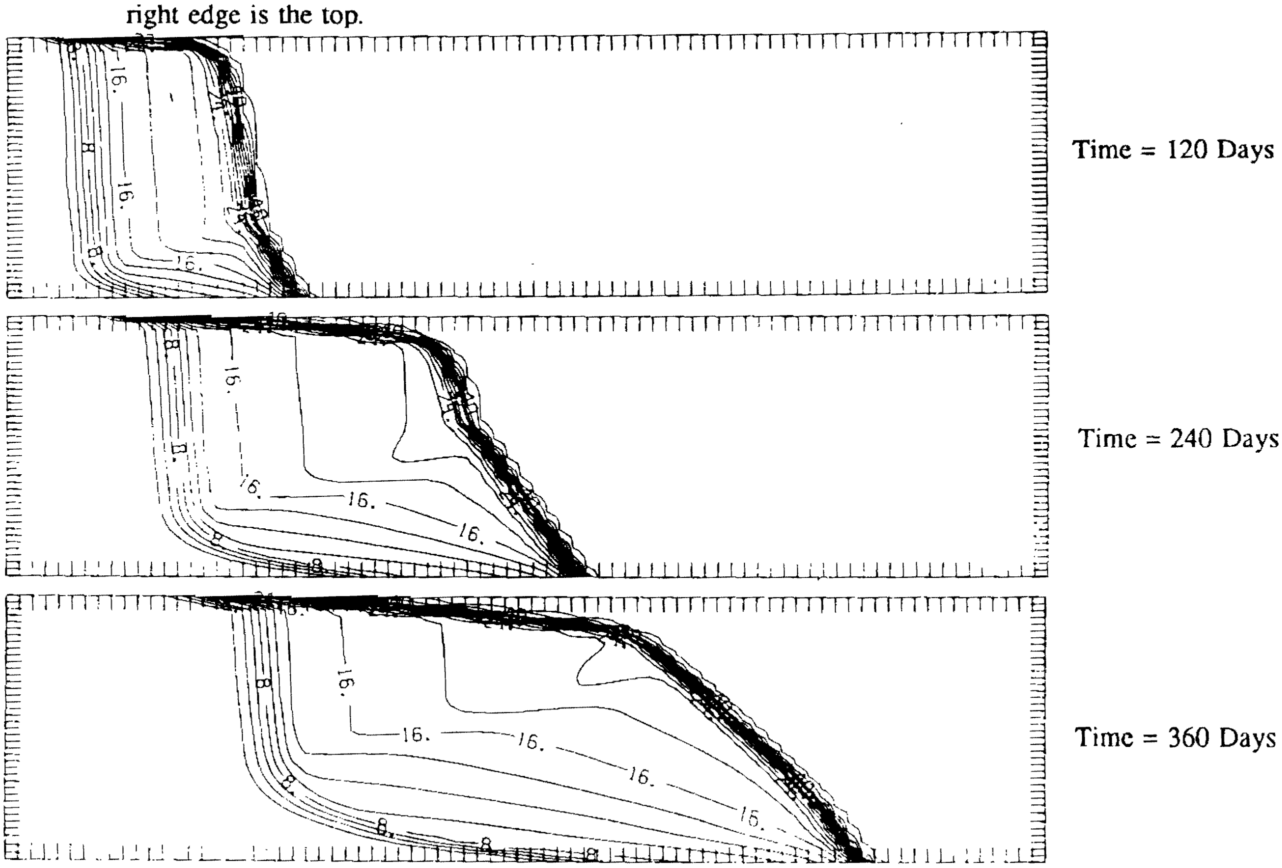
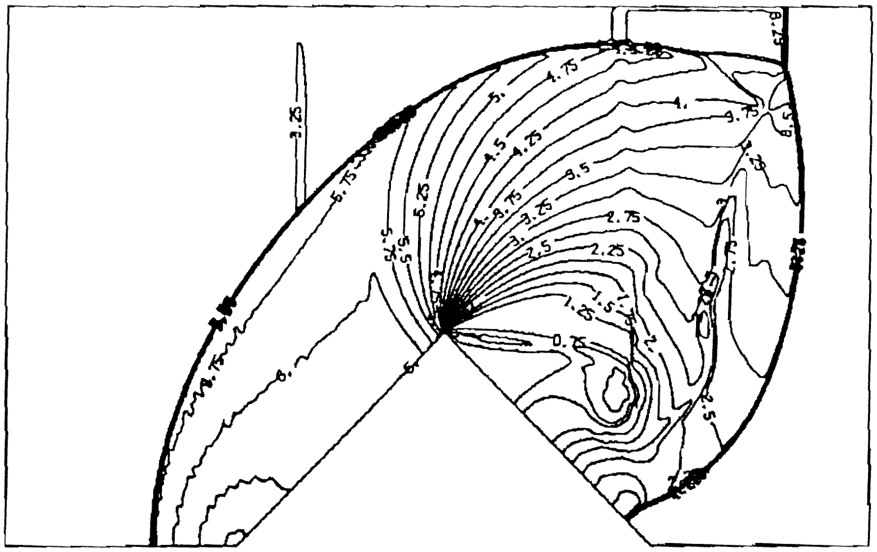


Fig.2. Contour of gas component density for two-dimensional waterflood on 240×80 grid.

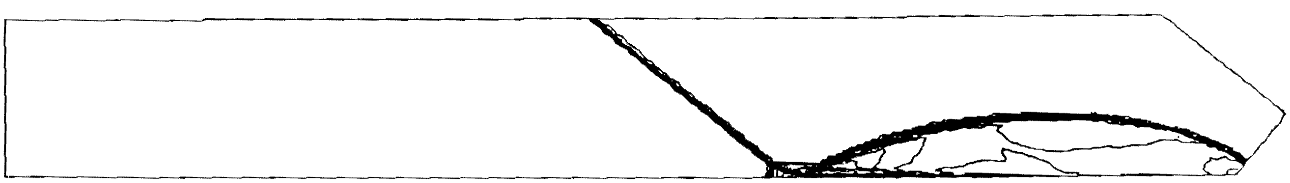


Density contours
440×220 grid

Experimental Interferogram
of Zhang and Glass

Fig.3. Comparison of numerical solution with experiment for shock diffraction over a half-diamond cylinder. Shock Mach number is 2.45.

Density contours at early time



Stationary density contours on expanding grid

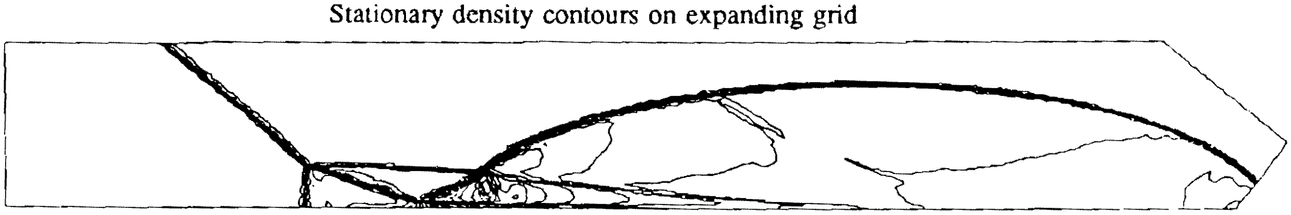


Fig. 4. Self-similar ramp reflection on a moving quadrilateral grid.

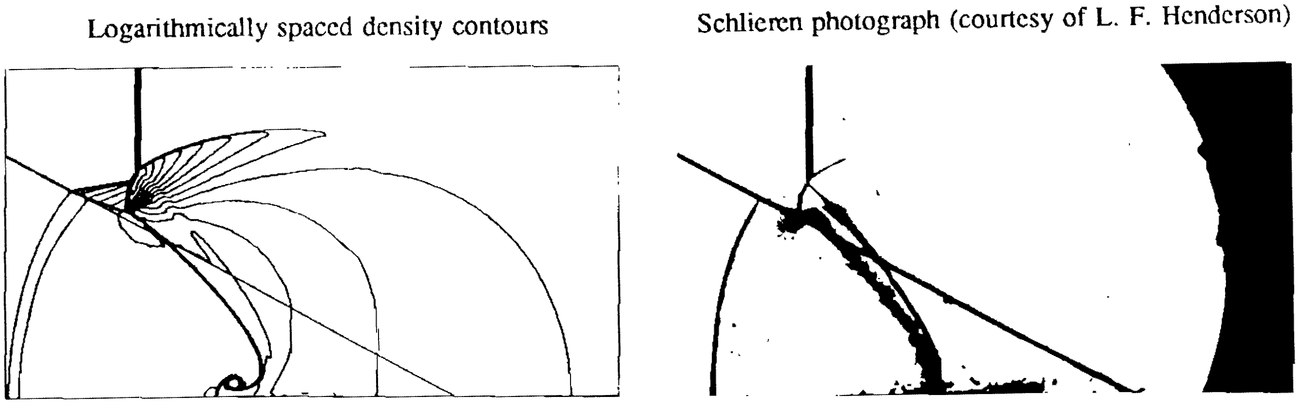


Figure 5. Shock refraction from an oblique material interface.

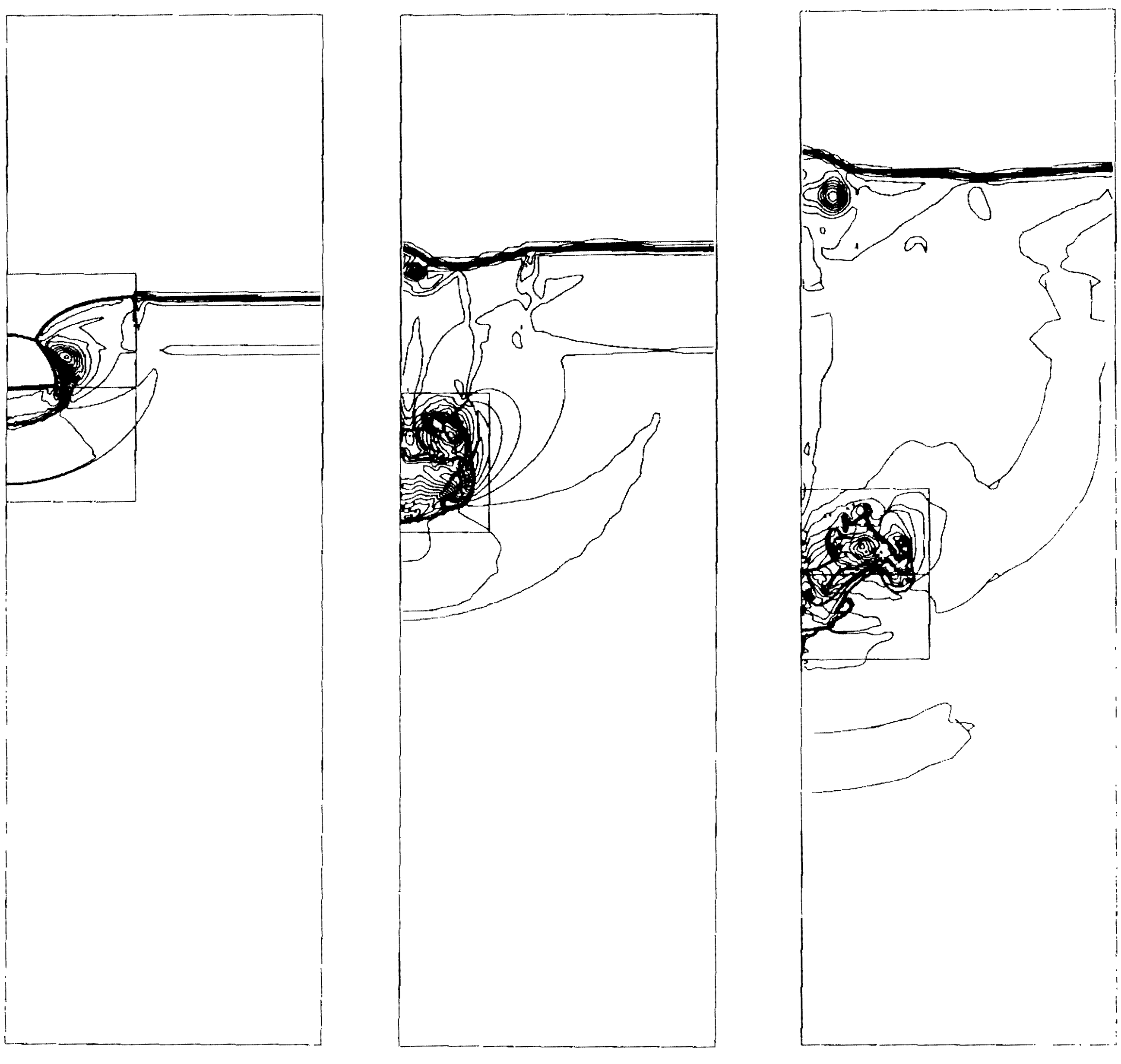


Fig.6. Refraction of a planar shock by a dense spherical cloud